AD-A009 737

TOWARDS SHORTER WAVELENGTH LASERS AND BREAKING THE 1000 ANGSTROM BARRIER-III (RECOMBINATION)

A. W. Ali, et al

Naval Research Laboratory
Washington, D. C.

March 1975

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 3015	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD-A009 737
4. TITLE (and Subtitle) TOWARDS SHORTER WAVELENGTH LASERS AND BREAKING THE 1000 ANGSTROM BARRIER-III (RECOMBINATION)		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. W. Ali and W. W. Jones		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem N01-33 Project RR-011-07-41-5072
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Office of Naval Research Arlington, Virginia 22217		12. REPORT DATE March 1975
		13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Short wavelength lasers
Recombination

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The physics of recombination in a multi-ionized and dense plasma is discussed. Emphasis is given to such a medium as a potential source for VUV and x-ray lasers. Results of some model calculations for hydrogen and C^{+5} ($\lambda = 34$ Å) are presented.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

Introduction	1
The Basic Physics of Recombination Plasmas	3
Hydrogen-like Energy Level and Gain	6
Inversion and Recombination Relaxation Time	7
Excited Level Population	11
The Electron Temperature and the Average Charge State in a Plasma	13
Quantitative Estimates of Inversion Density and Gain	16
References	18

Preceding page blank

Towards Shorter Wavelength Lasers and
Breaking the 1000 Å Barrier-III
(Recombination)

1. Introduction

Discussions on the possibility of VUV and X-Ray lasers can be found in the literature as long as ten years ago[1,2]. The interest in the subject, however, was enhanced in 1967 when the possibility of photon pumping as a possible mechanism for the realization of VUV ($\lambda < 1000$) and X-Ray lasers[3] ($\lambda < 10$ Å) was discussed. Meanwhile, there have arisen a variety of proposals and schemes on this subject. The electron and photon pumping mechanisms were discussed in Ref. (4) and the bound-continuum emission as a source for short wave lasers was discussed in Ref. (5). In this report the recombining plasma is considered as a medium for the amplification of short wave lasers ($\lambda < 1000$ Å).

Gudzenko and Shelepin[6] were first to point out the possibility of radiation amplification in a recombining plasma. Their calculations for an optically thin hydrogen plasma based on non-equilibrium plasma conditions showed inversion and reasonable gain in the visible. Their method basically follows the standard work of Bates, et al.[7], who have calculated the steady state values of the excited state densities for optically thin and thick hydrogen and hydrogen like plasmas. These calculations[7] were carried out for given electron densities and temperatures with the motivation to obtain the total recombination coefficient and the deviation of the excited states from their equilibrium (Saha) densities. An optically thick calculation for laser purposes was also carried out by Gordiets et al.[8], for a hydrogenic plasma. Bohn[9], recently, have calculated inversion densities and gain coefficients for hydrogen like ions with emphasis on short wave lasers. His calculations[9]

Note: Manuscript submitted February 14, 1975.

follow the standard Bates et al.[7] method. A similar calculation for hydrogen plasma is carried out by Skorupski and Suckewer[10] with emphasis on the effect of heavy particle (hydrogen atom) collisions on the excited state populations.

For hydrogen-like ions the energy spacing between discrete levels increases as Z^2 (where Z is the nuclear charge). Therefore, shorter wavelength lasers ($\lambda < 1000$ Å) can be realized within a few times ionized hydrogen-like plasmas. For example the n = 3 to n = 2 transition in hydrogen (where n and m are the quantum numbers) corresponds to 6562 Å, which is the well known H_α line. In He II the same transition is 1640 Å while in Li III it corresponds to 729 Å.

A time dependent solution for the inversion density and the gain coefficient with a self consistent treatment of radiation, we believe, is a step forward in the recombination plasma model calculations. This has been carried out by Jones and Ali[11]. It should be pointed out, however, that a time dependent calculation for He II with emphasis on 1640 Å line has been carried out by Gudzenko, et al.[12] for a helium plasma where the effect of charge exchange between an excited Helium ion state and a Helium atom is considered. They have also considered[13] some aspects of cw operation of a recombining plasma. Inversion between the resonance and the ground-state of hydrogen in a two level atom approximation has been considered analytically by Peyrand and Peyrand[14].

One important aspect of the recombination plasma as a source of radiation amplification is that the electron cooling must be faster than the plasma recombination. Some aspects of this has been considered by Pert and Ramsden[15]. On the other hand, Irons and Peacock[16] indicate the presence of some population inversion in C^{+5} ion in an expanding laser produced plasma.

In this report, the physics of recombination as a source for short wave lasers ($\lambda < 1000$ Å) is considered and some results of a model calculations are presented.

2.0 The Basic Physics of Recombination Plasmas

The process of recombination which signifies the relaxation phenomenon in an ionized medium has been of great interest to astrophysicist and plasma spectroscopists. Radiation, as line spectra and continuum, accompany the recombination process. A quantitative measurement of these spectra enables one to describe the physical state of a plasma.

The astrophysicists have generally investigated plasmas with a low degree of ionization, where collisional processes ($N_e \ll 10^8$ cm^{-3}) are not significant. Radiative capture and cascade are the dominant processes which could describe the line intensities from gaseous nebulae, for example. Baker and Menzel[17,18] originally approached this problem by calculating the line intensities from a gaseous nebulae for two well known cases: plasmas 1) optically thin and 2) optically thick to Lyman series radiation. Application of their techniques to astrophysical problems with further refinements have been carried out by numerous investigators: i) Burgess[19], taking explicit account of the orbital angular momentum and 2) Seaton[20] taking into consideration the three-body recombination for higher quantum levels.

It is of interest at this juncture to point out that based on purely radiative recombination and cascade ($N_e \ll 10^8$ cm^{-3}) one can not obtain inversion in a hydrogen or hydrogen-like plasma. This is because direct recombination favors lower quantum levels. For example, for hydrogen the recombination coefficient $\alpha_r(n)$ to level n is[21]

$$\alpha_r(n) \propto \frac{1}{n} \frac{1}{\sqrt{T}} \quad , \tag{1}$$

when the mean thermal energy of electrons, kT, is small compared to the ionization energy. However, when the thermal energy is larger than the ionization energy, then[21]

$$\alpha_r(n) \propto \frac{1}{n^3} \frac{1}{T^{3/2}} \quad , \tag{2}$$

again favoring the lower levels. Furthermore, detailed calculations by Drawin and Emard[22] for the instantaneous population densities of excited states show lack of inversion. For example, for $T \cong 4000^{o}K$ and $N_e \cong 10^{13}\ cm^{-3}$ the departure coefficient from Saha-Equilibrium for $n = 4$ and $n = 3$ are 6.5×10^{-2} and 8.25×10^{-2} respectively, indicating that no inversion is possible. It should be pointed out, however, that even if inversion occurs at such a low density, gain is insufficient for laser action.

For dense plasmas, however, with higher degrees of ionization, the recombination is essentially collisional and three-body recombination dominates. The pioneering work of Bates et al.[7] with their extension of recombination to plasmas with higher electron densities and thereby including three-body processes, have been followed by numerous investigators with interest in plasma physics and diagnostics of laboratory plasmas. These calculations have either solved a large set of rate equations[7,23-25] which describe the population densities of excited states, or are based on a variational model[26,27] which yields the total recombination coefficients. Therefore, for laser purposes, with a recombining, high density plasma, the method of Bates et al.[7] is essentially the starting point, since it can solve for the population densities of the excited states and their method has been followed by many[6,9,10].

On the other hand, the three-body recombination coefficient, $\alpha_3(n)$, has a dependence on the quantum number favoring the higher levels. This coefficient can be obtained from the ionization rate coefficient, S_n, through the detailed balance (see reaction 3)

$$N_e + N_e + N^z \rightleftarrows N^{z-1}(n) + N_e \quad , \tag{3}$$

where N_e is the electron density, N^z is the ion density, Z times ionized, and $N^{z-1}(n)$ is the population density of level n within the ion which is Z - 1 times ionized.

The ionization rate coefficient can be expressed as

$$S_n = \frac{8.0 \times 10^{-8} n^2}{Z^2 E_H \sqrt{T_e}} \exp\left(-\frac{Z^2 E_H}{n^2 T_e}\right) , \tag{4}$$

where E_H is the ionization energy of hydrogen. Here T_e and E_H are in units of eV. This expression is obtained using the excitation rate coefficient[4] given by Seaton with an oscillator strength of unity and an average gaunt factor of $\frac{1}{2}$. Bates, et al.[7] using an expression of this kind indicate that (4) is reasonable within a factor of 2 for helium. Using detailed balance one obtains

$$\frac{N_e N^z}{N^{z-1}(n)} = \frac{S_n}{\alpha_3(n)} \quad . \tag{5}$$

However, the left hand side of (5) is the well known Saha distribution[28] which can be expressed as

$$\frac{N_e N^z}{N^{z-1}(n)} = 3.02 \times 10^{21} \frac{g^z}{n^2} \; T_e^{3/2} \exp\left(-\frac{Z^2 E_H}{n^2 T_e}\right) , \tag{6}$$

where g^z is the statistical weight of the ground state of N^z. Using (4) - (6) one obtains

$$\alpha_3(n) = 2.65 \times 10^{-27} \frac{n^4}{Z^2 E_H g^z T_e^2} \quad (\mathrm{cm}^6/\mathrm{sec}) \quad . \tag{7}$$

It is obvious that the three body recombination rate coefficient for a given Z is proportional to n^4 and the lower the temperature, the higher the recombination rate. Furthermore, for the same quantum level, the recombination rate coefficient is inversely proportional to Z^2.

2.1 Hydrogen-like Energy Level and Gain

The wavelength for a transition between two hydrogen-like energy levels n and m is

$$\lambda_{nm} = \frac{1.24 \times 10^{-4}}{Z^2 E_H} \frac{n^2 m^2}{n^2 - m^2} \quad , \tag{8}$$

where E_H is in units of eV. Consider, for example, a $3 \rightarrow 2$ transition. This transition is 6562 Å in hydrogen, 1640 Å in He II and 729 Å in Li III. Thus the breaking of the 1000 Å barrier can be achieved in a recombining plasma where Li is three times ionized. Shorter wavelength lasers can be obtained if one considers transitions with $\Delta n > 1$ in the above ions.

The inversion density between two discrete levels u and ℓ with densities N_u and N_ℓ, respectively is $\frac{N_u}{g_u} - \frac{N_\ell}{g_\ell}$, where g_u and g_ℓ are the corresponding statistical weights. The gain coefficient, in cm^{-1} is[4]

$$\alpha = 10^{-12} A_{u\ell} \cdot \left(N_u - N_\ell \frac{g_u}{g_\ell} \right) \cdot \frac{\lambda_{u\ell}^4}{\Delta\lambda_{u\ell}} \quad , \tag{9}$$

where $A_{u\ell}$ is the spontaneous transition rate and $\Delta\lambda_{u\ell}$ is the width of the transition. One can express the gain in Eq. (9) for a hydrogen-like ion in terms of the quantum numbers of the levels, the oscillator strength of the transition and the nuclear charge. This can be done by utilizing the expression[28] for the transition rate $A_{u\ell}$ i.e.

$$A_{u\ell} = 4.3 \times 10^7 \frac{g_\ell}{g_u} f_{u\ell} Z^4 E_H^2 \left(\frac{1}{\ell^2} - \frac{1}{u^2} \right)^2 \quad . \tag{10}$$

Thus, using (8) and (10) into (9) one obtains

$$\alpha = \frac{5.5 \times 10^{-23}}{\Delta\lambda_{u\ell}} \left[N_u - N_\ell \frac{g_u}{g_\ell} \right] \frac{f_{u\ell}}{Z^4} \cdot \frac{\ell^2}{u^2} \frac{u^4 \ell^4}{(u^2 - \ell^2)^2} \quad . \tag{11}$$

In most cases of interest, $\Delta\lambda_{u\ell}$, is Stark broadened and must be calculated for each electron density. However, when $\Delta\lambda_{u\ell}$ is Doppler broadened it is proportional to $\lambda_{u\ell}$ and thus to $\frac{1}{Z^2}$. Therefore, the gain in terms of quantum levels as given in (11) would be proportional to $\frac{1}{Z^2}$. Using the generally accepted scaling for hydrogen-like ions i.e. $N_e = n_e Z^7$ and $T_e = Z^2\theta$, where n_e and θ_e are the electron density and temperature for hydrogen, one obtains the scaled gain to be proportional to Z^7. This scaling for a Doppler broadened line and $T_i = T_e$ is

$$\alpha_z = \alpha_H \cdot Z^7 \cdot \sqrt{\frac{M_z}{M_H}} \quad \frac{1}{A_H} \quad . \tag{12}$$

2.2 Inversion and Recombination Relaxation Time

When a plasma is completely ionized i.e., when the constituents are completely stripped ions and electrons, the plasma will relax unless the electrons are continuously heated by an external source. After a short time the plasma reaches a quasi equilibrium state. During this early stage of recombination one is expected to see laser action due to the filling of the higher states. This is because, the three body recombination clearly favors the higher quantum levels (see Eq. 7). To calculate the total recombination rate it is instructive to point out that there is a quantum level[29], n^*, above which all discrete excited levels are in equilibrium with the free electrons. Furthermore[29], the number of transitions transferred from continuum downward thru this level, equals the number of transitions upward from below the same level. This quantum state is given by

$$n^* = \sqrt{\frac{Z^2 E_H}{T_e}} \quad ,$$

and can be arrived at by the following argument. Since transitions among the nearest neighbors are the most important transitions, one obtains

$$N_{n+1} Y_{n+1,n} = N_{n-1} X_{n-1,n} \quad , \tag{13}$$

where $X_{n,n'}$ is the excitation rate from n to n$'$ and $Y_{n',n}$ is the corresponding de-excitation rate. Equation (13) implies that

$$\frac{g_n}{g_{n-1}} \exp\left(-\frac{E_n - E_{n-1}}{T_e}\right) = 1 \quad , \tag{14}$$

or $E_n \simeq T_e$ and $n^* \simeq \sqrt{\frac{Z^2E_H}{T_e}}$. Thus, the total recombination rate is the sum of Eq. (7) from n = 1 to n = n* which results in

$$(\tau_3)^{-1} = 2.65 \times 10^{-28} (Z^2E_H)^{3/2}(T_e)^{-4.5}N_e^2 \quad , \tag{15}$$

where $g_z = 2$ was utilized. This result differs (for Z = 1) from corresponding results in Refs. 29 and 30 by a factor of 2.35 and 1.5, respectively. These differences arise primarily because of the choice of the cross sections used. However, the more rigorous result derived by Pitaevskii and by Gurevich and Pitaevskii (see Ref. 30) is

$$\alpha_3^T = \frac{4\pi(2\pi)^{1/2}}{9} \frac{e^{10}Z^3}{\sqrt{m}(kT)^{9/2}} \log \Lambda \quad , \tag{16a}$$

which reduces, with log $\Lambda \simeq 1$, to

$$\alpha_3^T = \frac{8.75 \times 10^{-27} Z^3}{(T_e)^{4.5}} \quad (cm^6/sec) \tag{16b}$$

The range of applicability of this relation is for $Z^2E_H \gg T_e$ or $n^* \gg 1$. For hydrogen the validity of this relation is for $T_e \leq 0.26$ eV.

However, a total three body recombination rate coefficient will have an entirely different temperature dependence and indeed even a different value if one utilizes the ionization rate coefficients reported by McWhirter[31]. Consider the following expression for the ionization coefficient of species Z - 1, with ionization energy $E^{z-1} = Z^2 E_H$.

$$S^{z-1} = \frac{9 \times 10^{-6}}{(E^{z-1})^{3/2}} \frac{\left(T_e/E^{z-1}\right)^{1/2}}{4.88 + \frac{T_e}{E^{z-1}}} \exp\left(\frac{-E^{z-1}}{T_e}\right) , \qquad (17a)$$

or

$$S^{z-1} = \frac{1.1 \times 10^{-5}}{(E^{z-1})^{3/2}} \frac{\left(T_e/E^{z-1}\right)^{1/2}}{6 + \frac{T_e}{E^{z-1}}} \exp\left(\frac{-E^{z-1}}{T_e}\right) . \qquad (17b)$$

These expressions are valid[31] over a wide range i.e., $.02 < \frac{T_e}{E^{z-1}} < 100$, which clearly encompasses the region of the validity of (16b). The corresponding three body recombination rate coefficient, using (17a) is

$$\alpha_3^T = \frac{1.5 \times 10^{-27}}{(E^{z-1} T_e)^{3/2}} \frac{(T_e/E^{z-1})^{1/2}}{4.88 + \frac{T_e}{E^{z-1}}} , \qquad (18)$$

which for $T_e/E^{z-1} \ll 1$ clearly has an entirely different temperature and Z dependence, that is $\alpha_3^T \propto Z^{-6} \; T_e^{-1}$. For a comparison between (18) and (16b) let $T_e = 1$ eV, $Z = 2$. The rate coefficients will be 10^{-27} cm^6/sec and 70×10^{-27} cm^6/sec respectively. This discrepancy, which is even larger than two orders of magnitude in certain instances will have a profound result on resonance line transitions if one considers how fast the hydrogenic ion becomes a helium-like ion.

Starting from completely stripped ions, the early stage of the recombination results in inversion and hence laser action among the highly excited and discrete states of the hydrogen-like ions (see Fig. 1

for inversion densities in hydrogen). Laser action generally terminates after times short compared to the life times of the transitions. These times are of the order of the electron de-excitation times of the upper laser levels or times when equilibrium conditions are attained. The time required for level $n (n \neq 1)$ to be in equilibrium with the higher levels $n' > n$ and the free electrons is equal to the inverse of the sum of the excitation rates to all higher levels, or approximately, to the next higher level. For a hydrogen-like ion, the excitation rate coefficient from level n to n′ is[4]

$$X_{nn'} = \frac{1.6 \times 10^{-5}\, f_{nn'}}{\Delta E_{nn'} \sqrt{T_e}} \exp\left(-\frac{\Delta E_{nn'}}{T_e}\right) \cdot g_{nn'} \quad , \qquad (19a)$$

where T_e and $\Delta E_{nn'}$ are in eV. Assuming a unit Gaunt factor reduces and using $\Delta E_{nn'} \doteq \frac{2\, Z^2 E_H}{n^3}$, and $\Sigma\, f_{nn'} \simeq 0.5\, n$ (see Ref. 28), (19a) becomes

$$\sum X_{nn'} = \frac{4 \times 10^{-6}\, n^4}{Z^2 E_H \sqrt{T_e}} \exp\left(-\frac{2\, Z^2 E_H}{n^3 T_e}\right) \quad . \qquad (19b)$$

Thus, the time for level n to reach equilibrium with the upper levels is

$$\tau_n = \frac{2.5 \times 10^5}{n^4 N_e} Z^2 E_H \sqrt{T_e} \exp\left(\frac{2\, Z^2 E_H}{n^3 T_e}\right). \qquad (20)$$

This expression (20), should be used when, in general, there are many discrete levels above the level of interest. On the other hand when only a few levels are discrete (see the discussion on the reduction of the ionization limits in section 2.3) then the de-excitation time from one level to another will determine the termination of the laser action. Furthermore, for equilibration down to the ground state of the ion, the de-excitation rate of the resonance level determines the required time for LTE to set in. In this case the time of interest is

$$\tau_{21} = \frac{1}{Y_{21} N_e} \quad , \tag{21}$$

where $Y_{n'n}$ is the electron impact de-excitation rate coefficient from level n' to $n(n' > n)$ and it can be obtained from (19) via detailed balance.

2.3 Excited Level Population

The laser action in a recombining plasma occurs between a pair of excited states of an atom or an atomic ion. It can also take place between an excited state and the ground state of the atom or the atomic ion. Therefore, a knowledge of the population densities of these states is essential for the laser power density calculations, the requirements for an efficient laser and the optimum conditions for lasing. The population density of a level can be obtained, in general, either by following the techniques of Bates et al.[7] which gives the steady state solutions for a set of rate equations, or by solving the rate equations as a function of time. The steady state solution ignores the early time spikes of lasers which may occur as the levels are being filled before they eventually attain a steady state. Therefore, the steady state underestimates the rate at which the charge disappears at early times where the levels are being filled. Furthermore, a steady state solution for a generalized hydrogenic system can not account for the loss of the N^{z-1} stage of ionization. However, a time dependent model which also solves[11] for the radiation densities connecting any two pairs of levels will self consistently describe the actual time development of the levels. In fact a model which also considers the time variation of the electron temperature due to the inelastic collisions and the dynamic expansion of the plasma is much superior to the steady state solution. The processes affecting the population density of an excited level are three-body recombination, radiative recombination, spontaneous decay and stimulated emission from higher levels, electron superelastic de-excitation of higher levels, radiative absorption from lower levels and electron impact excitation from lower levels. All the above processes increase the

population density of a given level. In contrast, the processes which decrease the population density of a level are: radiative decay, stimulated emission, electron impact de-excitation to lower levels, electron impact excitation which raise the electron to higher levels, ionization by electron impact, photoabsorption and ionization.

Thus, in principle one must solve for a large number of excited states. However, the number of states is limited because of the reduction in the ionization limit in a plasma. The last discrete level, n_d, is[28]

$$n_d = \left(\frac{Z^2 E_H}{\Delta E^{z-1}} \right)^{1/2} \quad , \tag{22}$$

where ΔE^{z-1} is the reduction in the ionization limit and is given by

$$\Delta E^{z-1} = \frac{Z_e^2}{\rho_D} \quad . \tag{23}$$

In (23) ρ_D is the Debye radius which is defined[32] as

$$\rho_D = 6.9 \left(\frac{T}{N'} \right)^{1/2} \quad , \tag{24}$$

where T is the electron temperature in degrees Kelvin and[28]

$$N' = N_e + \sum_z Z^2 N^z \quad . \tag{25}$$

In general, however, most plasmas are not completely ionized. Therefore, the population densities of the excited states of interest may be affected by other processes which must be included. Among these; is collisions of heavy particles (atoms or ions) with the excited state. These collisions may excite, de-excite and cause charge transfer to or from the level of interest.

2.4 The Electron Temperature and the Average Charge State in a Plasma

A knowledge of the electron temperature and its evolution is of considerable interest in the recombination calculations. For example the cooling of the electrons increase the three-body recombination rate to a given level, at a faster rate $\left(\sim \frac{1}{T_e^2}\right)$ than the increase in the level de-excitation rate by electrons $\left(\sim \frac{1}{\sqrt{T_e}}\right)$. Furthermore, for high Z ions, the cooling depletes the ground state at a faster rate $\left(\frac{1}{T^{4.5}}\right)$; thereby increasing the inversion density and the duration for n = 2 to 1 transitions. Therefore the time history of the electron temperature should be known for the plasma of interest. In this section a semi-quantitative approach is taken to estimate the electron temperature and the charge state in a plasma in relation to the energy input, especially for the laser produced plasmas. High temperature and high density plasmas are obtained in several ways. The most notable and novel is the laser produced plasma[33,34]. For a review article on this subject see Ref. 35. One of the important aspects of this novel approach is that high electron densities $n_e \gg 10^{19}$ cm^{-3} and high electron temperatures $T \simeq$ keV can be achieved. However, plasmas with high electron densities can also be obtained through shock tubes[36-38], coaxial accelerators[39] and by high energy electron beams[40]. It is interesting to point out at this juncture that the state of the plasma in shock tubes e.g., whether LTE is established or not have been a center of controversy among the investigators[36-39]. It would be desirable to see if population inversion occurs in these transient plasmas and whether or not this mechanism may be the reason for the disagreement[36-39]. The authors plan to review this in the near future.

In a laser produced plasma, the electrons gain energy from the incident laser beam through inverse bremstrahlung (in a collisional plasma) and cool through ionization, excitation, electron-electron and electron-ion collisions. The three body recombination heats the electrons in a recombining plasma, however, as the plasma expands into a vacuum, the electrons cool adiabatically. Thus, the electron temperature can be described provided all relevant processes are considered. The energy

density, $\bar{w}$, deposited in a target with charge Z to produce N_e electrons with temperature T_e can be written as

$$\bar{w} = \frac{3}{2} N_e T_e + \frac{3}{2} T_i \sum_{z=1}^{\prime} N_{z-1} + \sum_{z=1}^{\prime} N_{z-1} E_{z-1}$$

$$+ \sum_{z=1}^{\prime} N_{z-1} I_{z-1} \quad , \tag{26}$$

where T_i is the average ion temperature (in eV), E_{z-1} is the excitation energy of the internal states of species, Z - 1 and I_{z-1} is the ionization energy of the same species. The prime on the summations indicates the last stage of ionization desired. For example, in a completely stripped helium, the summation is up to Z = 2. Thus, for the case of completely stripped ions, the energy density is

$$\bar{w} = \frac{3}{2} N_e T_e \left[+ \frac{T_i}{ZT_e} + \frac{2}{3\ ZT_e} \sum_{z=1}^{\prime} E_{z-1} + \frac{2}{3\ ZT_e} \sum_{z=1}^{\prime} I_{z-1} \right] . \tag{27}$$

This coupled with the degree of ionization and average charge state calculations can, in principle, give the electron temperature phenomenologically. On the other hand, the temperature evolution can be calculated from the first electron stripping to the last stages of ionization in a laser produced plasma from the laser absorption principle.

The average charge state in a plasma can be calculated following Raizer[30,41] by assuming that the ion densities and the ionization potentials are continuous functions of the ionic charge Z. This can be expressed as

$$N_z = N_{z-1} + \frac{dN}{dZ} \quad ,$$

or

$$\frac{N_z}{N_{z-1}} = 1 + \frac{1}{N_{z-1}} \frac{dN}{dZ} \quad . \tag{28}$$

However, $\frac{dN}{dZ}$ is zero at the peak of the ionization. Therefore, for a collision dominated plasma expression (28) can be related to Saha equation (see Eq. 6 for n = 1) i.e.

$$N_e \doteq 6 \times 10^{21} \, T_e^{3/2} \exp\left(- \bar{I}/T_e\right) \quad , \tag{29}$$

where T_e is in units of eV, the statistical weights are set to unity, and $\bar{I}$ is the average ionization energy. Thus for a given electron density and temperature one finds the average ionization energy and thus the average charge state using values of the ionization potentials[42]. The charge state, Z, thus obtained should correspond to the average charge $\bar{Z}$ by simply adding[30,41] one half to Z.

As an example, consider carbon ionized six times which implies that $\bar{I} = E_H = 490$ eV and assume an electron density of 10^{22} cm^{-3}. Using (29) one obtains $T_e \approx 80$ eV. This can be coupled with (27) to give the required or the minimum energy input. Assuming that $T_e \simeq T_i$, the energy density can be approximated for the above condition by

$$\bar{w} \doteq \frac{3}{2} N_e T_e \left[1.2 + \frac{1}{ZT_e} \sum_{z=1}^{6} I_{z-1} \right] ,$$

or

$$\bar{w} \doteq 6 \times 10^5 \text{ J/cm}^3 \quad .$$

This energy should then be delivered in times short compared to the recombination time and for lasers in the resonance state, the energy should be delivered in times short compared to the electron de-excitation time of the resonance level. For carbon, this time is $\sim 10^{-12}$ sec (see Eq. 21). This implies that 6×10^{17} watts/cm^3 is needed for one cubic

centimeter. However, if one considers a carbon sample of 1 cm long and 1 μ wide then the power required will be 6×10^9 watts.

The assumption that the electron temperature is equal to the ion temperature over estimates the energy input. The equipartition time between electrons and ions is[22]

$$\tau_{ei} = 3 \times 10^7 \frac{A_z}{Z^2 N_z} T_e^{3/2} \quad , \tag{30}$$

where A_z is the atomic number and T_e is in eV. For $T_e \simeq 80$ eV, Z = 6 and $N_e = 10^{22}$ cm^{-3} one obtains $\tau_{ei} = 4 \times 10^{-12}$ sec. This time is longer than the duration of inversion.

3. Quantitative Estimates of Inversion Density and Gain

In this section quantitative estimates of inversion density and gain for resonance line transitions in hydrogen like ions will be given.

Consider for simplicity a completely stripped carbon with T_e = 100 eV and $N_e = 10^{22}$ cm^{-3}. Using equations 22-25 one obtains $n_d \simeq 4$ which implies that only few discrete levels are observable. Furthermore, the upper level of the resonance line i.e. n = 2 will be in equilibrium with the higher discrete levels and the free electrons. This can be obtained using the criterion for equilibration[28] which states that

$$N_e \geq \frac{7 \times 10^{18} Z^3}{n^{17/2}} \left(\frac{T_e}{Z^2 E_H} \right)^{1/2} \quad . \tag{31}$$

However, level 2 is not in equilibrium with the ground state and inversion is established between n = 2 and n = 1 at the early stage of recombination simple because the recombination rate to n = 2 is larger than the corresponding rate to n = 1. In addition, level n = 1 will be depleted towards the formation of the helium-like ion and the establishment of the collisional ionization equilibrium.

The inversion density can be obtained by using the equilibrium value for n = 2. Using Eq. (6) one obtains $N_2 \simeq 4 \times 10^{19}$ cm^{-3}, which is approximately equal to the inversion density. The gain, on the other hand,

is $\alpha \cong 28\ cm^{-1}$, where equation (9) is utilized with the Stark width of the transition. The full Stark width[43] is

$$\Delta\lambda \simeq \frac{12}{\pi} (n_i^2 - n_f^2) N_p^{2/3} \cdot \frac{\lambda^2}{c} \quad . \tag{32}$$

Using Eq. (32) one obtains $\Delta\lambda \simeq 1$ Å, which is much larger than the Doppler width (assuming that $T_i = T_e$). The above analytic estimates agree very well with the detailed[11] code calculations.

However, it should be recognized that the duration of inversion is short and is of the order of the electron de-excitation rate of level 2. This process is responsible for the equilibration of level 1 with the higher levels and thus will terminate any laser action.

Figure (2) shows the inversion density and the gain for n = 2 to n = 1 transition in hydrogen-like carbon ($\lambda \simeq 34$ Å) for several initial electron temperatures. These results are from a detailed model calculations [11]. However, for $T_e \simeq 100$ eV, the simple quantitative analysis, given in this section, predicts results in good agreement with the detailed calculation.

4. Acknowledgement

The work of one of us (W.W. Jones) is supported by N01-33.

REFERENCES

1. A. L. Schawlow, Quantum Electronics and Coherent Light, Townes and Miles, Eds., Academic Press, New York (1964).

2. L. Gold, Proc. 3rd Int. Conference on Quantum Electronics, Paris (1963) 2, 1155 (1964).

3. M. A. Duguay and P. M. Rentzepis, Appl. Phys. Letters 10, 350 (1967).

4. A. W. Ali, "Towards Shorter Wavelength Lasers and Breaking the 1000 Å Barrier - I, (Electron and Photon Pumping)," NRL Memorandum Report 2792 (1974).

5. A. W. Ali, "Towards Shorter Wavelength Lasers and Breaking the 1000 Å Barrier - II, (Bound-Continuum Emission)," NRL Memorandum Report 2863 (1974).

6. L. I. Gudzenko and L. A. Shelepin, JETP, 18, 998 (1964). L. I. Gudzenko and L. A. Shelepin, Soviet Phys. Doklady 10, 147 (1965).

7. D. R. Bates, A. E. Kingston and R. W. P. McWhirter, Proc. Royal Soc. (London) A267, 297 (1962); 270A, 155 (1962).

8. B. F. Gordiets, L. I. Gudzenko and L. A. Shelepin, J. Quart. Spect. Rad. Transfer 8, 701 (1968).

9. W. L. Bohn, Appl. Phys. Lett. 24, 15 (1974).

10. A. A. Skorupski and S. Suckewer, Phys. Lett 46A, 473 (1974).

11. W. W. Jones and A. W. Ali, "A Computer Code for Time Dependent Recombination Laser," NRL Memo Rept. 2999 (1975).

12. L. I. Gudzenko, Yu. K. Zemtsov and S. I. Yakovenko, Soviet Physics JETP 12, 244 (1970).

13. L. I. Gudzenko, M. V. Akezlin and S. I. Yakovlenko, Soviet Physics, Tech. Physics 18, 1218 (1973).

14. J. Peyrand and N. Peyrand, J. Appl. Phys. 43, 2993 (1972).

15. G. J. Pert and S. A. Ramsden, Optics Comm. 11, 270 (1974).

16. F. E. Irons and N. J. Peacock, J. Phys. B. Atom. Mol. Phy. 7, 1109 (1974).

17. J. G. Baker and D. H. Menzel, Ap. J. 88, 52 (1938).

18. Selected Papers on Physical Processes in Ionized Plasmas, D. H. Menzel, ed., Dover Publications Inc., New York (1962).

19 A. Burgess, Month. Not. Roy. Ast. Soc. 118, 477 (1958).

20. M. J. Seaton, Month Not. Roy. Ast. Soc. 127, 177 (1964).

21. D. R. Bates and A. Dalgarno, in "Atomic and Molecular Processes," D. R. Bates, ed. Academic Press, New York (1962).

22. H. W. Drawin and F. Emard, "Instantaneous Population Densities of the Excited Levels of Hydrogen Atoms, Hydrogen Like Ions and Helium Atoms in Optically Thin and Thick non LTE Plasmas," DPH-PFC/SPP EURCEAFC 534 (1970).

23. R. W. P. McWhirter and A. G. Hearn, Proc. Phys. Soc. (London) 82, 641 (1963).

24. V. A. Abramov, Soviet Phys., High Temp. 3, 18 (1965).

25. D. W. Norcross and P. M. Stone, J. Quant. Spect. Rad. Trans. 8, 665 (1968).

26. S. Byron, R. C. Stabler and P. I. Bortz, Phys. Rev. Lett. 8, 376 (1962).

27. B. P. Curry, Phys. Rev. A1, 166 (1970).

28. H. R. Griem, Plasma Spectroscopy, McGraw-Hill, New York (1964).

29. E. Hinnov and J. G. Hirschberg, Phys. Rev. 125, 795 (1962).

30. Ya. B. Zeldovich and Yu. P. Raizer, "Physics of Shock Waves and High Temperature Hydrodynamic Phenomena," Vol. I. Academic Press, New York (1966).

31. R. W. P. McWhirter, "Plasma Diagnostic Techniques," Huddleston and Leonard Eds. Academic Press, New York (1965).

32. L. Spitzer, Jr., "Physics of Fully Ionized Gases," Interscience Publishers Inc., New York (1956).

33. B. C. Fawcett, A. H. Gabriel, F. E. Irons, N. J. Peacock and P. A. H. Saunders, Proc. Phys. So (London) 88, 1051 (1966).

34. N. G. Basov, V. A. Boika, Yu. P. Voinov, E. Ya. Kononov, S. L. Mandelshtam, and G. V. Skilzkov, JETP Letters 5, 141 (1967).

35. P. Mulser, R. Sigel and S. Wikowski, Physics Reports, Physics Letters 6c, 189 (1973).

36. E. A. McLean, C. E. Fanoff, A. C. Kolb and H. R. Griem, Phys. Fluids 3, 843 (1961).

37. R. C. Isler and D. E. Kerr Phys. Fluids 8, 1176 (1965).

38. K. K. Eckerle and R. W. P. McWhirter, Phys. Fluids 9, 81 (1966).

39. T. N. Lie, A. W. Ali, E. A. McLean and A. C. Kolb, Phys. Fluids 10, 1545 (1967).

40. P. S. P. Wei, W. M. Leavens, and J. L. Adamski, J. Appl. Phys. 45, 2153 (1974).

41. Y. P. Raizer, JETP 9, 1124 (1959).

42. T. A. Carlson, C. W. Nestor, Jr., Neil Wasserman and J. D. McDowell, Atomic Data 2, 63 (1970).

43. H. R. Griem,"Broadening of Spectral Lines by Plasmas," Academic Press, New York (1974).

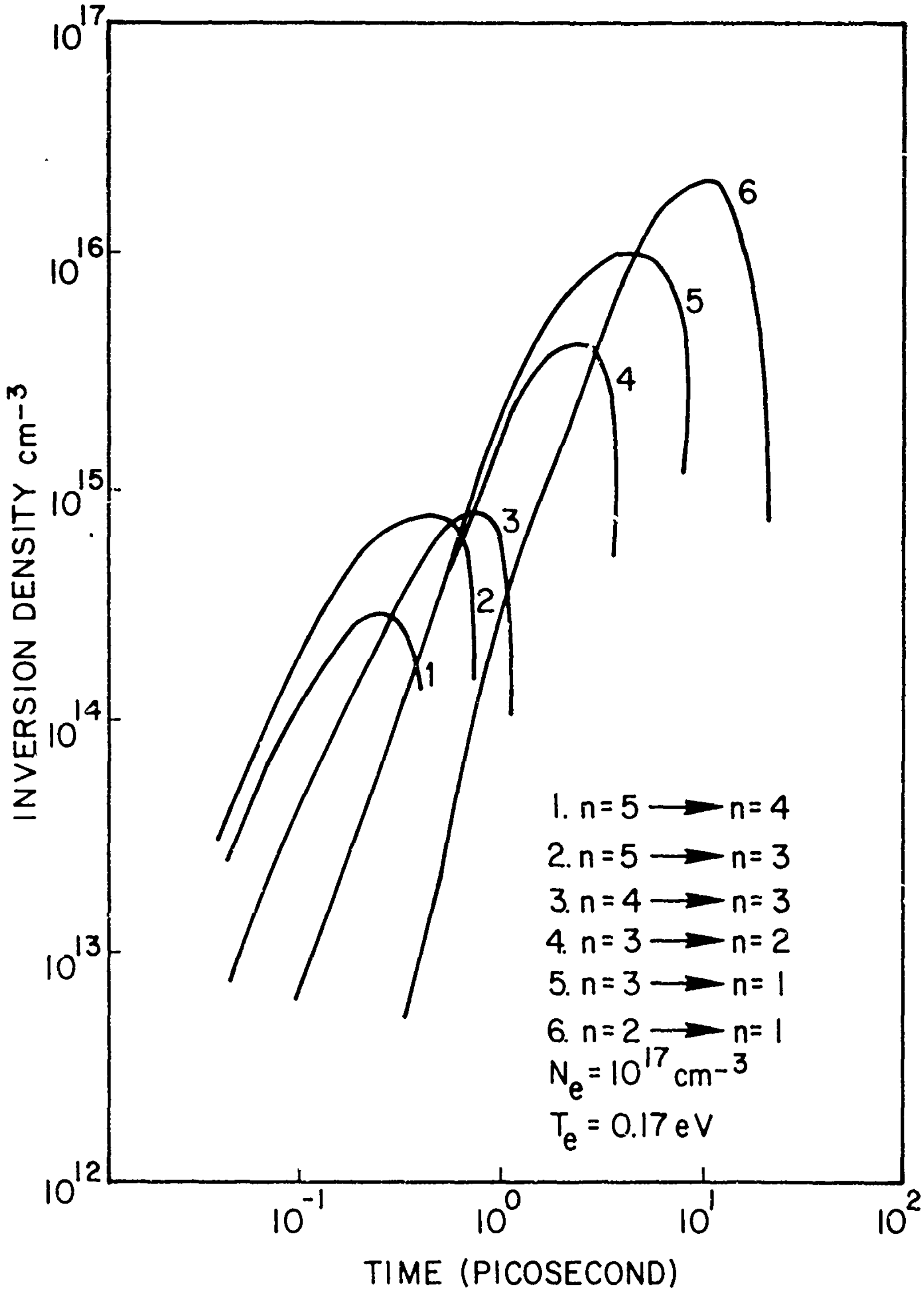

Fig. 1 — The inversion densities for several transitions in hydrogen are shown as a function of time

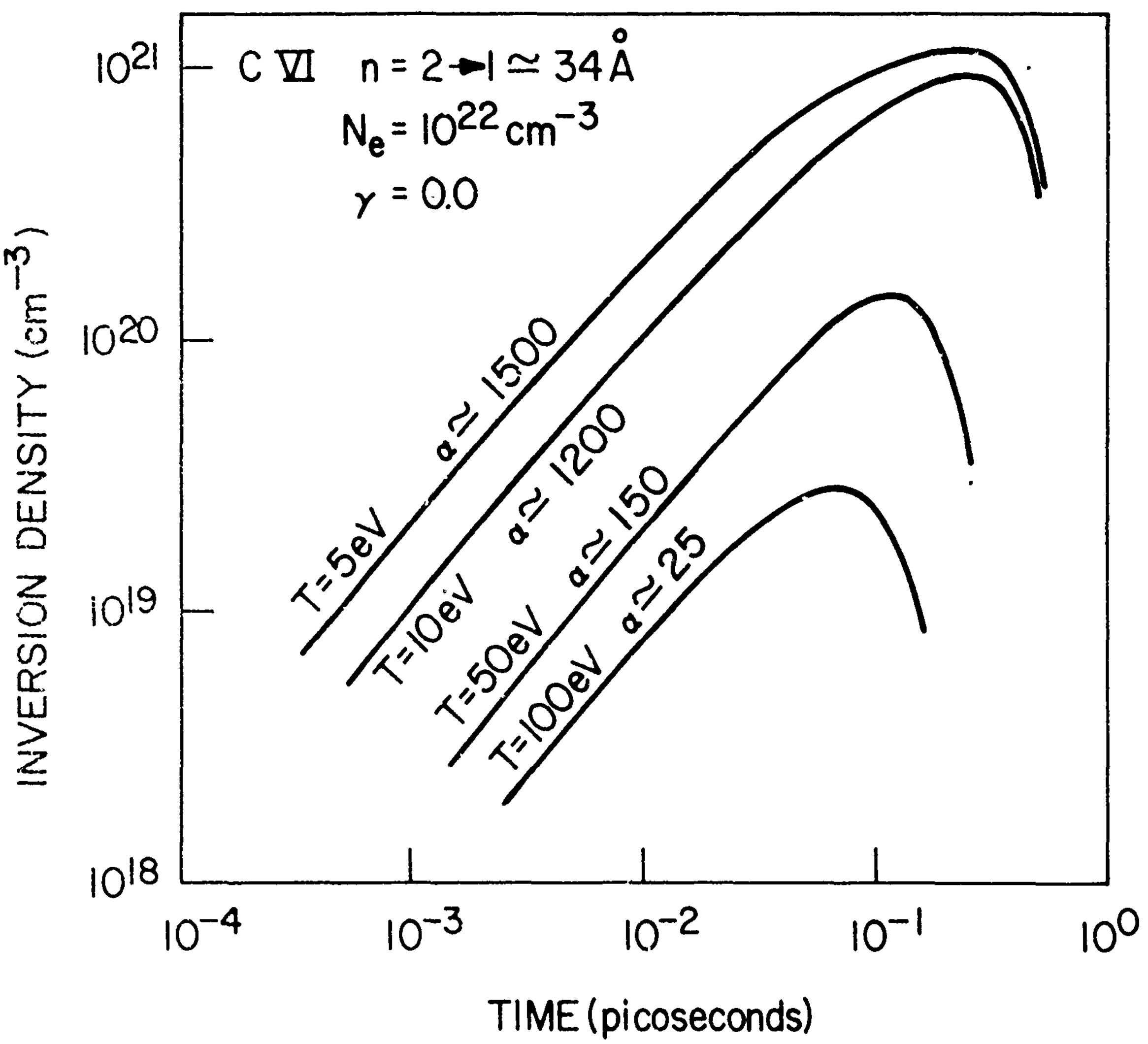

Fig. 2 — The inversion density for the resonance transition in C^{+5} is presented for several electron temperatures